-2

CORNELL AERONAUTICAL LABORATORY

0

CAL/CM-441

LIFT OF FINITE SWEPT WINGS WITH
SUPERSONIC LEADING EDGES

J. M. Gwinn

20 January 1948 Copy No. 26
CAL-1-A-4

B U F F A L O, N E W Y O R K

26

CORNELL AERONAUTICAL LABORATORY
Buffalo, New York

LIFT OF FINITE SWEPT WINGS WITH SUPERSONIC LEADING EDGES

By

J. M. Gwinn
Fundamental Aerodynamics Branch

20 January 1948
CAL-1-A-4

Report No. CAL/CM-441
Copy No. 26

CORNELL AERONAUTICAL LABORATORY
Buffalo, New York

TABLE OF CONTENTS

CORNELL AERONAUTICAL LABORATORY
Buffalo, New York

CAL/CM-441

ABSTRACT

The lift coefficient and center of pressure of swept finite wings are calculated. The product, $A\beta$, the aspect ratio times the Mach parameter $(\beta = \sqrt{M^2-1})$ and the ratio, k/β, the tangent of the sweep angle divided by the Mach parameter, are the determining variables of the problem. Center of pressure and lift coefficient have been plotted against k/β for various values of $A\beta$.

SYMBOLS

From Snow's paper

x = spanwise axis

y = vertical axis

z = axis in direction of flow

δ = angle leading edge makes with x axis

μ = Mach angle

$\cos\beta = \tan\delta / \tan\mu$

α = angle of attack

$C_{L_\infty} = 4\alpha \tan\mu$

w = perturbation velocity

From Puckett's paper

y = spanwise axis

z = vertical axis

x = axis in flow direction

σ = angle leading edge makes with y axis

θ = Mach angle

$\beta = \sqrt{M^2 - 1}$

$k = \tan\sigma$

$n = k/\beta$

General

b = semi-span

c = chord

$A = \frac{2b}{c}$ = aspect ratio

x', y' = oblique coordinates corresponding to x, y.

CORNELL AERONAUTICAL LABORATORY
Buffalo, New York

CAL/CM-441

INTRODUCTION

Snow[1] has used Busemann's method of conical fields to determine pressure distributions for a class of wing planforms. It is the purpose of this paper to extend this class to finite swept wings with supersonic leading edges, having neither taper nor rake. Rather than attempt to bring the reader up to date, the writer has treated the work as an addendum to Snow's paper.

DISCUSSION OF THE PROBLEM

The problem is first to determine the pressure distributions in three regions on the wing and then to sum them over the wing to obtain lift and center of pressure (see Fig. 1). Region C is inside of the Mach cone originating at the apex of the wing. Region A is outside of apex and tip Mach cones. Region B is inside of the tip Mach cone. Since the pressure distributions in regions A and C have already been calculated

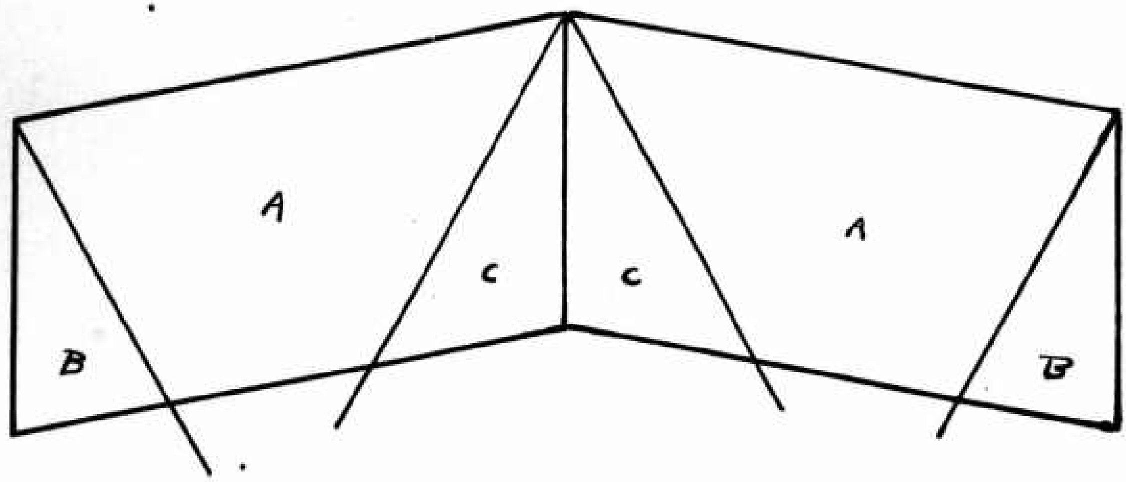


FIGURE 1

by Snow, his results are used. It remains to find the pressure distribution in region B. This is done first, and the pressure distributions are then integrated to obtain lift and center of pressure.

TIP REGION (B)

The work here will be developed analogous to that for the rectangular wing. Notation is that of Snow.

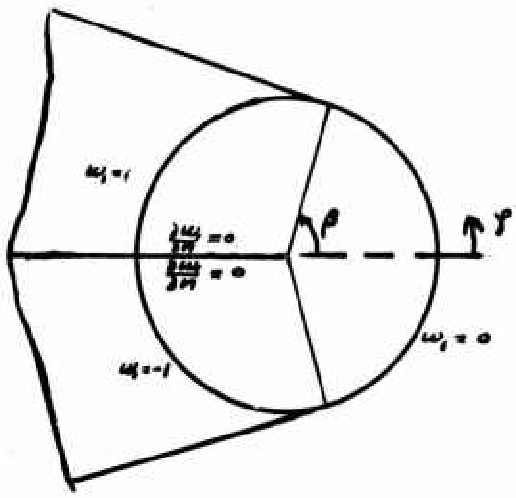


FIGURE 2

Boundary conditions

$R = A \; (r = 1)$

$$\omega_1 = 0 \; ; \quad -\beta < \varphi < \beta$$

$$\omega_1 = 1 \; ; \quad \beta < \varphi < \pi$$

$$\omega_1 = -1 \; ; \quad -\pi < \varphi < -\beta$$

$\varphi = \pm \pi$

$$\frac{\partial \omega_1}{\partial n} = 0$$

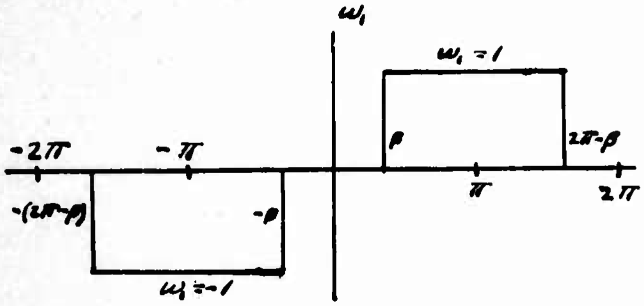
(Note that $\beta = \pi/2$ corresponds to the rectangular wing.) The general solution to the potential problem is

$$\omega_1 = \sum_0^\infty A_n \, r^{n+1/2} \sin\left(n+\tfrac{1}{2}\right)\varphi \, ,$$

the presence of the half-odd integers being necessary to satisfy the boundary conditions. For R = A,

$$\sum_0^\infty A_n \sin\left(n+\tfrac{1}{2}\right)\varphi = \text{value of the boundary}$$

where A_n is the Fourier coefficient for a sine series of period 4π. Note that the boundary conditions have been extended to the region



$\pi < |\varphi| < 2\pi$

such that ω_1 remains an odd function.

Then,

$$A_n = -\frac{1}{2\pi}\int_{-(2\pi-\beta)}^{-\beta} \sin(n+\tfrac{1}{2})\varphi \, d\varphi$$

$$+\frac{1}{2\pi}\int_{\beta}^{2\pi-\beta} \sin(n+\tfrac{1}{2})\varphi \, d\varphi$$

$$= \frac{4}{\pi}\,\frac{\cos(n+\frac{1}{2})\beta}{n+\frac{1}{2}}$$

and

$$\omega_1 = \frac{4}{\pi}\sum \frac{r^{n+\frac{1}{2}}}{n+\frac{1}{2}}\cos(n+\tfrac{1}{2})\beta \, \sin(n+\tfrac{1}{2})\varphi$$

The value of ω_1 on the wing corresponds to $\varphi = \pi$. The expression for ω_1 becomes, after simplification,

$$\omega_1 = \frac{2}{\pi}\sum_0^{\infty}\frac{\cos\frac{\beta}{2}(-r)^{n+\frac{1}{2}}}{n+\frac{1}{2}}\cos n\beta - \frac{2}{\pi}\sum_0^{\infty}\frac{\sin\frac{\beta}{2}(-r)^{n+\frac{1}{2}}}{n+\frac{1}{2}}\sin n\beta$$

Differentiation with respect to r yields

$$\frac{\partial \omega_1}{\partial r} = \frac{2}{\pi \sqrt{r}} \left[\cos\frac{\beta}{2} \sum_0^{\infty} (-r)^n \cos n\beta - \sin\frac{\beta}{2} \sum_0^{\infty} (-r)^n \sin n\beta \right]$$

$$= \frac{2}{\pi \sqrt{r}} \sum_0^{\infty} \left[\cos\frac{\beta}{2}\, \mathcal{R}\left\{ (-re^{i\beta})^n \right\} - \sin\frac{\beta}{2}\, \mathcal{I}\left\{ (-re^{i\beta})^n \right\} \right]$$

Noting that $\sum_0^{\infty} (-re^{i\beta})^n$ is a geometric series, one obtains

$$\frac{\partial \omega_1}{\partial r} = \frac{2}{\pi \sqrt{r}} \left[\cos\frac{\beta}{2}\, \mathcal{R}\left(\frac{1}{1+re^{i\beta}} \right) - \sin\frac{\beta}{2}\, \mathcal{I}\left(\frac{1}{1+re^{i\beta}} \right) \right]$$

After real and imaginary parts are separated, integration yields

$$\omega_1 = \frac{2}{\pi} \tan^{-1} \sqrt{\frac{2r(1+\cos\beta)}{1-r}}$$

or,

$$\omega_1 = \frac{2}{\pi} \sin^{-1} \sqrt{\frac{\frac{R}{A}(1+\cos\beta)}{1+\frac{R}{A}\cos\beta}}$$

The quantity under the radical defines a new R/A (say, $\frac{R'}{A}$). It is seen that for $\beta = \frac{\pi}{2}$, $\frac{R'}{A} \longrightarrow \frac{R}{A}$.

On the wing,

$$\frac{R}{A} = -\frac{x}{z \tan\mu}$$

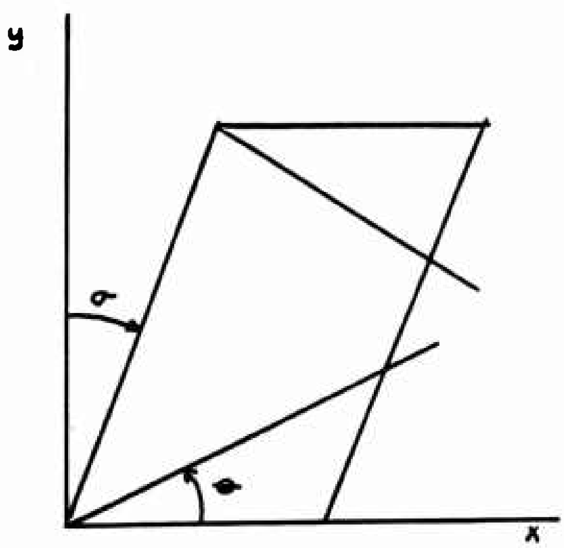
Thus,

$$\omega_1 = \frac{2}{\pi} \sin^{-1} \sqrt{\frac{\frac{-x}{2\tan\mu}\left(1+\cos\beta\right)}{1 - \frac{x}{2\tan\mu}\cos\beta}}$$

$$= \frac{2}{\pi} \sin^{-1} \sqrt{\Theta}$$

LIFT CORRECTION FOR FINITENESS

The notation of Puckett makes the integration of the pressure coeff- cients (to be calculated later) simpler.



Puckett	Snow
x	$z + \frac{s}{2}\tan\delta$
y	$\frac{s}{2} + x$
θ	$\pi - \mu$
σ	δ
b	$s/2$
k	$\tan\delta$
β	$\cot\mu$
n	$\cos\beta$
$n = k/\beta$	
$4\alpha/\beta$	C_{L_∞}

Under these transformations

$$\Theta = \frac{(b-y)(\beta+k)}{x-ky}$$

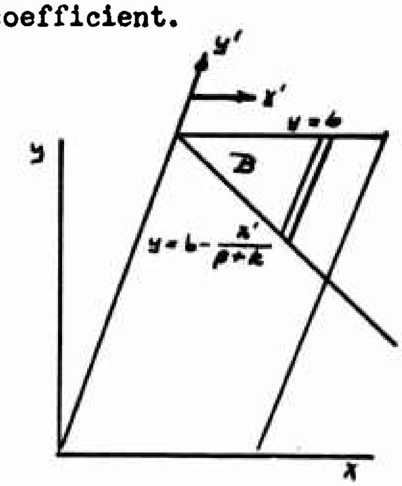
A second transformation

$$x' = x - ky$$
$$y' = y$$

further simplifies the integration. The expression for Θ now becomes

$$\Theta = \frac{(b-y)(\beta+k)}{x'}$$

C_{L_Δ} = correction to the infinite swept wing lift coefficient.



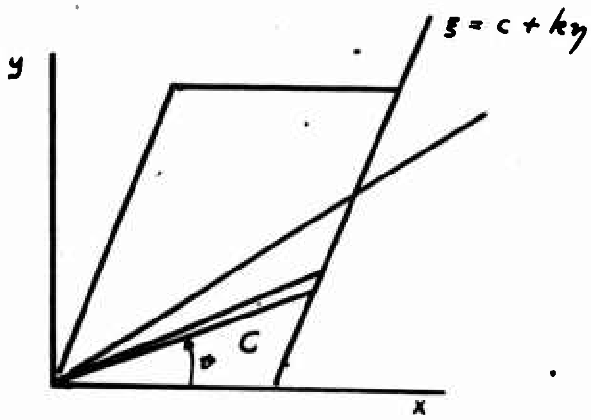
$$= \frac{4\alpha}{\beta\sqrt{1-n^2}} \iint_B (1-\omega_1)\, ds$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}} \int_0^c dx' \int_{b-\frac{x'}{\beta+k}}^{b} \cos^{-1}\sqrt{\Theta}\, dy'$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}} \frac{2}{\pi} \frac{1}{\beta+k} \int_0^c x'\,dx' \int_0^1 \cos^{-1}\sqrt{\Theta}\, d\Theta$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}} \frac{1}{2A\beta} \frac{1}{1+n}$$

LIFT CORRECTION FOR LEADING EDGE DISCONTINUITY



$$t = \frac{k\eta}{\xi}$$

$$dS = \frac{c^2}{2k}\frac{dt}{(1-t)^2}$$

(see appendix A)

Equation (21) of Snow's paper can be transformed to yield

$$C_{P_\Delta} = \frac{4\alpha}{\beta\sqrt{1-n^2}}\left[\frac{2}{\pi}\sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\right]$$

Note agreement between this and Pucketts equation (35)

Now

$$C_{L_\Delta} = \frac{1}{bc}\iint_C C_{P_\Delta}\,dS$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}}\frac{c^2}{2k}\frac{1}{bc}\frac{2}{\pi}\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\frac{dt}{(1-t)^2}$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}}\frac{1}{2A\beta}\frac{\frac{2}{\pi}\left[\frac{\pi}{2}n + \sqrt{1-n^2} - (1-2n^2)\frac{\sin^{-1}n}{n}\right]\frac{1}{1-n}}{1+n}$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}}\frac{1}{2A\beta}\frac{\Delta(n)}{1+n}$$

(see appendix B)

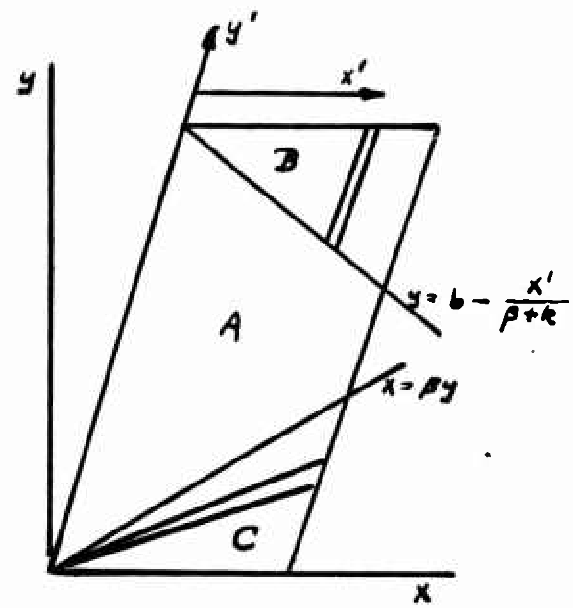
WING LIFT COEFFICIENT

$$C_L = \frac{4\alpha}{\beta\sqrt{1-n^2}}\left[1 - \text{Corrections from sweep and finiteness}\right]$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}}\left\{1 - \frac{1}{A\beta}\,\frac{1}{1+n}\,\frac{1+\Delta(n)}{2}\right\}$$

$$= \frac{4\alpha}{\beta\sqrt{1-n^2}}\left[1 - \frac{1}{A\beta} f(n)\right]$$

CENTER OF PRESSURE FROM APEX



$$(C_p)_A = \frac{4\alpha}{\beta\sqrt{1-n^2}}$$

$$(C_{p_\Delta})_B = \frac{4\alpha}{\beta\sqrt{1-n^2}}\,\frac{2}{\pi}\cos^{-1}\sqrt{\Theta}$$

$$(C_{p_\Delta})_C = \frac{4\alpha}{\beta\sqrt{1-n^2}}\,\frac{2}{\pi}\sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}$$

$$\frac{M_{y'}}{q} = \iint_{ABC} x' C_p \, dS$$

$$M_y' = \frac{c}{2}\left[bc\, q\, (C_p)_A\right] - \frac{2}{3}c\left[\text{lift decrement from } B/C\right]$$

$$= q\,\frac{4\alpha}{\beta\sqrt{1-n^2}}\left[\frac{bc^2}{2} - \frac{2}{3}\,bc^2\,\frac{f(n)}{A\beta}\right]$$

$$= \frac{\bar{X}'}{c} = \frac{\frac{1}{2} - \frac{2}{3}\frac{1}{A\beta}f(n)}{1 - \frac{1}{A\beta}f(n)}$$

$$\frac{M_x}{q} = \iint_{ABC} y\, C_p\, ds$$

$$\frac{M_x}{q} = \frac{4\alpha}{\beta\sqrt{1-n^2}}\int_0^c dx' \int_0^b \frac{b}{2}\,dy$$

$$-\frac{2}{\pi}\,\frac{4\alpha}{\beta\sqrt{1-n^2}}\int_0^c dx' \int_{b-\frac{x'}{\beta+k}}^{b}\left[b - \frac{x'}{2(\beta+k)}\right]\cos^{-1}\sqrt{\Theta}\; dy'$$

$$-\frac{1}{3}\,\frac{c^3}{k^2}\,\frac{2}{\pi}\,\frac{4\alpha}{\beta\sqrt{1-n^2}}\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\;\frac{t\,dt}{(1-t)^3}$$

$$\frac{M_x}{q} = \frac{4\alpha}{\beta\sqrt{1-n^2}}\, b c^2 \left\{ \frac{2b}{4c} - \frac{1}{4\beta(1+n)} + \frac{c}{12 b \beta^2 (1+n)^2} \right.$$

$$- \frac{1}{A\beta^2}\left[\frac{1}{4}\left(\frac{\sin^{-1} n}{n^2} - \frac{\sqrt{1-n^2}}{n}\right) - \frac{1}{2}(1-2n^2)\sin^{-1} n \right.$$

$$\left. \left. + n\sqrt{1-n^2} + \frac{3\pi}{8} n^2 \right] \frac{6}{(1-n)^2} \right\} \; . \quad \text{(see appendix C)}$$

$$\frac{\bar{y}}{c} = \frac{A}{1 - \frac{1}{A\beta} f(n)} \left\{ \frac{1}{4} - \frac{1}{4A\beta(1+n)} + \frac{1 - g(n)}{6A^2\beta^2(1+n)^2} \right\} ,$$

where $g(n)$ is the coefficient of $\frac{1}{A\beta^2}$ in the expression for M_x/q.

$$\bar{x} = \bar{x}' + k\bar{y} = \frac{c}{1 - \frac{1}{A\beta} f(n)} \left\{ \frac{1}{2}\left(1 - \frac{1}{2}\,\frac{n}{1+n}\right) \right.$$

$$\left. - \frac{1}{A\beta}\left[\frac{2}{3} f(n) - \frac{n(1-g)}{6(1+n)^2}\right] + \frac{n}{4} A\beta \right\}$$

$$\frac{\bar{x}}{c} = \frac{\frac{1}{2}\left(1 - \frac{1}{2}\,\frac{n}{1+n}\right) - \frac{1}{A\beta} F(n) + \frac{n}{4} A\beta}{1 - \frac{1}{A\beta} f(n)}$$

LIMIT ON η

The limiting value of η is determined from the boundary condition of the problem. Consider the case where the apex and tip Mach cones intersect on the wing. There are two possibilities; either the apex Mach cone cuts the wing tip or it does not. Comparison of Figs. 3 and 4 show that the boundary conditions are <u>not</u> fulfilled for the former (Fig. 4) since the solution for the apex Mach circle yields a non-zero value of ω in the wing plane between E and F. Therefore, the condition on η is that the apex Mach cone must not intersect the wing tip. Thus

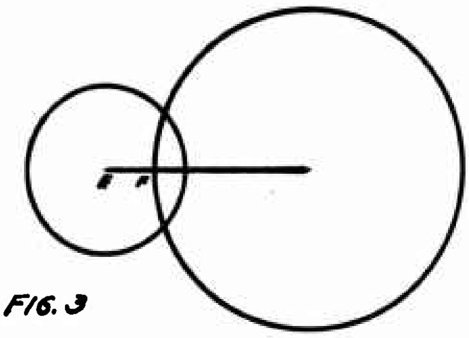


FIG. 3

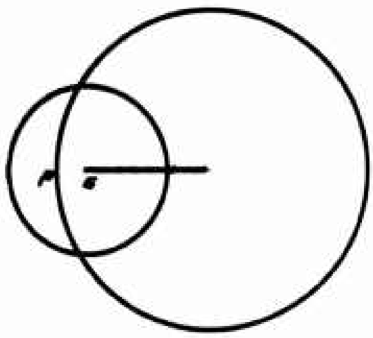
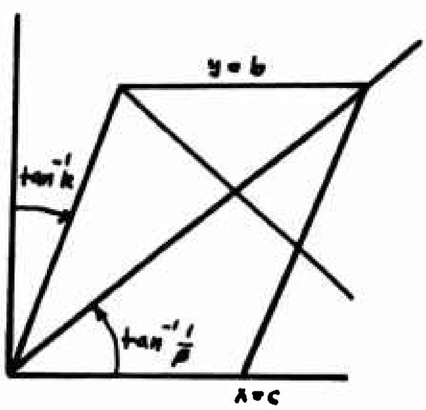


FIG. 4



$$\frac{b}{c + k b} = \frac{1}{\beta}$$

or

$$\frac{b/c}{1 + k\, b/c} = \frac{1}{\beta}$$

From which

$$n = 1 - \frac{2}{A\beta} .$$

This condition holds for $n \neq 0$. If $n = 0$, replace this by

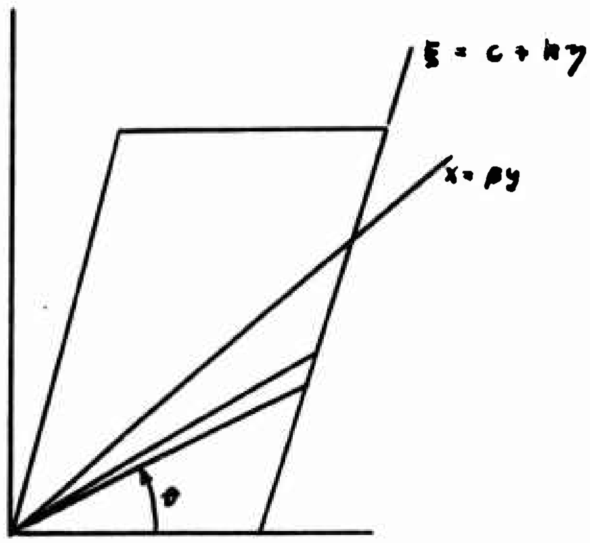
$$A\beta \geq 1 ,$$

a condition on the tip Mach cone.

RESULTS

The ratio, C_L/C_{L_∞}, the lift coefficient divided by the lift coefficient of an infinite wing normal to flow $\left(C_{L_\infty} = \frac{4\alpha}{\beta}\right)$ and the ratio, $\bar{x}/c$, the center of pressure divided by the chord, are plotted against n for various values of $A\beta$ in Figures 5 and 6.

APPENDIX A



$$ds = \frac{1}{2}\xi^2 \sec^2\theta \, d\theta$$

set $$t = \frac{k\eta}{\xi} = k\tan\theta$$

$$dt = k\,d\left(\frac{\eta}{\xi}\right) = k\sec^2\theta\,d\theta$$

Thus

$$ds = \frac{1}{2k}\xi^2\,dt$$

But

$$\xi = \frac{c}{1-t}$$

Finally

$$ds = \frac{c^2}{2k}\frac{dt}{(1-t)^2}$$

APPENDIX B

Integration of $\displaystyle\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\,\frac{dt}{(1-t)^2}$

set: $\sin\vartheta = \sqrt{\dfrac{n^2-t^2}{1-t^2}}$ $\qquad t=n:\ \vartheta=0$

$\qquad t=0:\ \vartheta=\sin^{-1}n$

$$t = \sqrt{\frac{n^2-\sin^2\vartheta}{1-\sin^2\vartheta}}$$

$$\frac{dt}{(1-t)^2} = \frac{-d\vartheta}{1-n^2}\left[\frac{\sin\vartheta\cos^2\vartheta}{\sqrt{n^2-\sin^2\vartheta}} + \sin 2\vartheta + \sin\vartheta\sqrt{n^2-\sin^2\vartheta}\right]$$

$$I \equiv \int_0^{\sin^{-1}n} \frac{\vartheta\sin\vartheta\cos^2\vartheta}{\sqrt{n^2-\sin^2\vartheta}}\,d\vartheta = \frac{\pi}{4}n^2 - III$$

$$II \equiv \int_0^{\sin^{-1}n} \vartheta\sin 2\vartheta\,d\vartheta = \frac{1}{2}\left[n\sqrt{1-n^2} - \sin^{-1}n\,(1-2n^2)\right]$$

$$III \equiv \int_0^{\sin^{-1}n} \vartheta\sin\vartheta\sqrt{n^2-\sin^2\vartheta}\,d\vartheta$$

$$I + II + III = \frac{n}{2}\left[\frac{\pi}{2}n + \sqrt{1-n^2} - (1-2n^2)\frac{\sin^{-1}n}{n}\right]$$

Thus

$$\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\,\frac{dt}{(1-t)^2}$$

$$= \frac{n}{2(1-n^2)}\left[\frac{\pi}{2}n + \sqrt{1-n^2} - (1-2n^2)\frac{\sin^{-1}n}{n}\right]$$

APPENDIX C

Integration of $\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\,\frac{t\,dt}{(1-t)^3}$

Using again the substitution

$$\sin\vartheta = \sqrt{\frac{n^2-t^2}{1-t^2}},$$

it is found that

$$\frac{t\,dt}{(1-t)^3} = -\frac{1}{(1-n^2)^2}\Big[4\sin\vartheta\cos^3\vartheta - 3(1-n^2)\sin\vartheta\cos\vartheta$$

$$+ 3\sin\vartheta\cos^2\vartheta\sqrt{n^2-\sin^2\vartheta} + \sin\vartheta\,(n^2-\sin^2\vartheta)^{3/2}\Big]$$

$$I \equiv 4\int_0^{\sin^{-1}n} \vartheta\sin\vartheta\cos^3\vartheta\,d\vartheta =$$

$$= -\sin^{-1}n\,(1-n^2)^2 + \frac{n}{4}(1-n^2)^{3/2} + 3/8\sin^{-1}n + \tfrac{3}{8}\,n\sqrt{1-n^2}$$

$$II \equiv 3(1-n^2)\int_0^{\sin^{-1}n} \vartheta\sin\vartheta\cos\vartheta\,d\vartheta$$

$$= -\frac{3}{2}(1-n^2)\Big[n^2\sin^{-1}n - \frac{\sin^{-1}n}{2} + \frac{n}{2}\sqrt{1-n^2}\Big]$$

$$III \equiv 3\int \vartheta\sin\vartheta\cos^2\vartheta\sqrt{n^2-\sin^2\vartheta}\,d\vartheta = \frac{3\pi}{16}n^4 - IV$$

$$IV \equiv \int \vartheta\sin\vartheta\,(n^2-\sin^2\vartheta)^{3/2}\,d\vartheta$$

Thus

$$\int_0^n \sin^{-1}\sqrt{\frac{n^2-t^2}{1-t^2}}\,\frac{t\,dt}{(1-t)^3}$$

$$= \frac{n^2}{2(1-n^2)^2}\left\{\frac{1}{4}\left(\frac{\sin^{-1}n}{n^2} - \frac{\sqrt{1-n^2}}{n}\right) - \frac{1}{2}(1-2n^2)\sin^{-1}n + n\sqrt{1-n^2} + \frac{3\pi}{8}n^2\right\}$$

REFERENCES

1. Snow, R. M., Application of Busemann's conical field method to thin wings of polygonal plan form, APL/JHU/CM-265, May 23, 1946.

2. Puckett, Allen E., Supersonic Wave Drag of Thin Airfoils, Journal of the Aeronautical Sciences, Vol. 13, Number 13, September 1946, pp. 475-484.

FINITE SWEPT WING LIFT COEFFICIENT

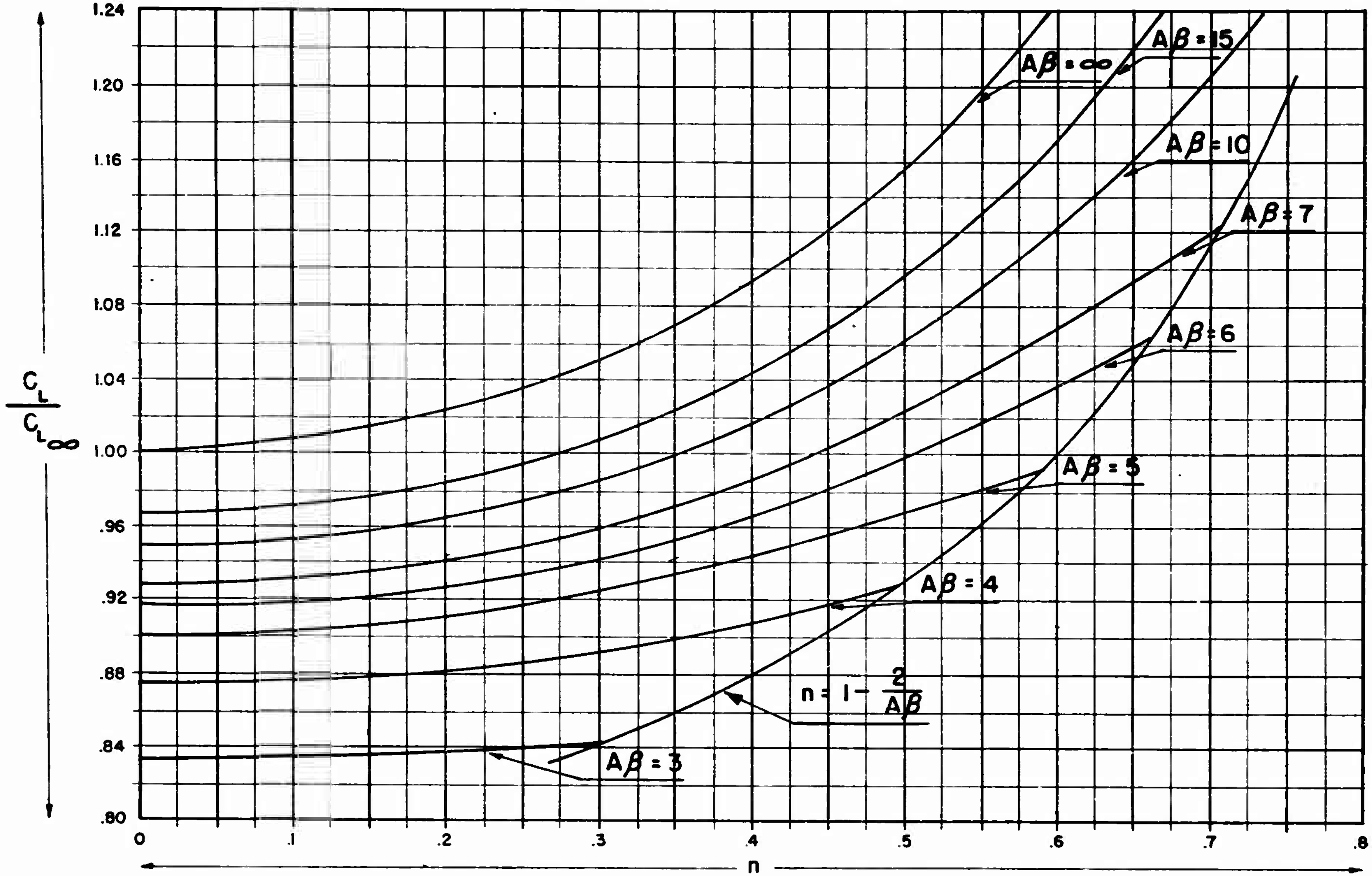


Fig. 5

FINITE SWEPT WING CENTER OF PRESSURE

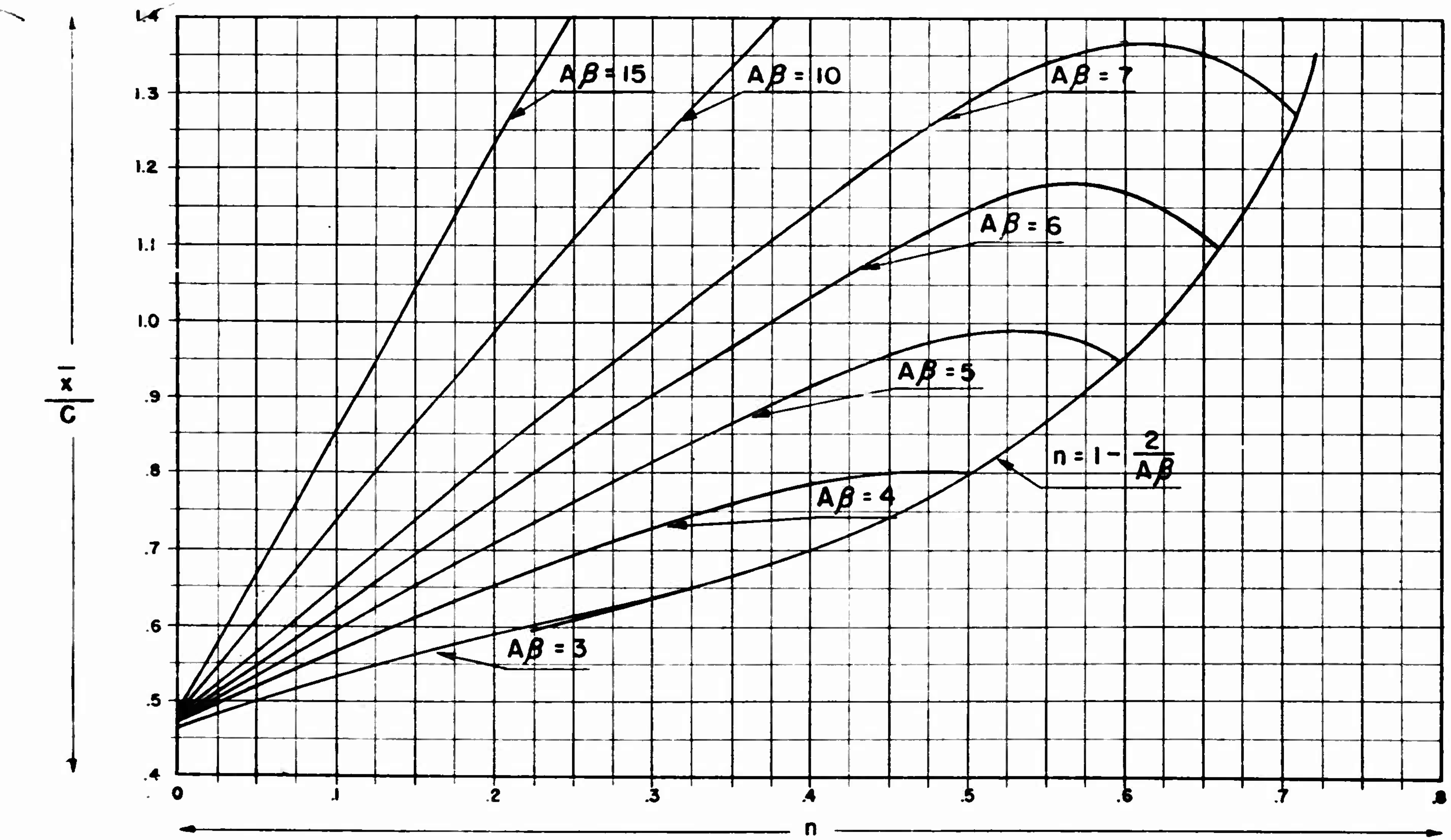


Fig. 6

TITLE: Lift of Finite Swept Wings with Supersonic Leading Edges AUTHOR(S): Gwinn, J. M. ORIGINATING AGENCY: Cornell Aeronautical Laboratory, Buffalo, N. Y. PUBLISHED BY: (Same)	ATI- 26934 REVISION (None) ORIG. AGENCY NO. CAL/CM-441 PUBLISHING AGENCY NO. (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Jan '48	Unclass.	U.S.	Eng.	28	diagrs

ABSTRACT:

Snow's class of thin wings of polygonal plan form, for which the pressure distribution has been determined by application of Busemann's conical-field method, is extended to include finite swept wings with supersonic leading edges having neither taper nor rake. The lift coefficient of swept finite wing divided by the lift coefficient of infinite wing normal to flow, and the center of pressure divided by the chord are plotted against the tangent of the sweep angle divided by the Mach parameter for various values of the aspect ratio times the Mach parameter.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Aerodynamics (2) SECTION: Wings and Airfoils (8) ATI SHEET NO.: R-2-6-82	SUBJECT HEADINGS: Wings, Swept-back - Lift (99306.4); Wings - Design (99160.6)